Walter Payton College Prep

Honors KAM-I/Geometry Course Outline

KAM-I/Geometry is a college-level course similar to 100- or 200-level courses in advanced geometry (first semester) and dynamical systems (second semester). The course begins with topics in Euclidean geometry beyond what is covered in the Geometry I curriculum: advanced constructions, triangle centers, the Fermat Point, the Nine Point Circle, etc. We discuss the algebra of transformations and include an in-depth look at synthetic inversion. In the second quarter we will go on to study axiomatic non-Euclidean Geometry, including Hilbert's axioms, Spherical, and Hyperbolic geometry. The second semester of KAM I will involve a thorough introduction to iteration, fractals, and chaos, including some advanced work with metric spaces and dynamical systems theory. Julia sets and the Mandelbrot sets will be studied at the conclusion of the course. Both semesters involve writing rigorous proofs, weekly problem sets, and a few out-of-class projects of various lengths.

Course Objectives

At the end of the course, students will be able to

- Complete advanced constructions involving multiple steps or ideas
- Prove theorems in advanced geometry
- Prove many "obvious" theorems using Hilbert's axioms
- Explain the relationship between Neutral, Spherical, and Hyperbolic geometry, giving examples of models for each and theorems true in one but not the other two
- Model situations with dynamical systems
- Explain and predict the long-term behavior of systems using fundamental theorems about iteration, dynamical systems, and chaos
- Give examples of fractals, explain how they are generated, and describe their properties qualitatively and quantitatively
- Describe iterations on the complex plane algebraically and geometrically, including stable and nonstable behavior
- Explain what the Mandelbrot and Julia sets are, how they are related to each other, how their properties relate to each other, and prove theorems about them
- Prove basic theorems of analysis using rigorous definitions of fundamental concepts

Course Topics Calendar

Weeks 1-4	Construction: Polyá's method, straightedge and compass constructions, problem-solving; Theorems about circles
Weeks 5-8	Advanced Geometry Theorems; Apollonius's Problem
Weeks 9-13	Transformations, Symmetry Groups, and Inversion
Weeks 14-18	Hilbert's Axioms, Neutral Geometry, Euclidean and Non-Euclidean Geometries
Weeks 19-21	Iteration of real-valued functions; phase space
Weeks 22-24	Fractals and fractal geometry; Hilbert (box) dimension
Weeks 25-27	Metrics and Fractal Space; theorems about convergence
Weeks 28-32	Chaos: definition of chaos, formal dynamical systems; isomorphisms between dynamical systems
Weeks 33-37	Complex dynamics; Hausdorff Space **(skip this)**; Mandelbrot and Julia sets

Resources Used

- Polyá, Mathematical Discovery vol. I
- Altshiller-Court, *College Geometry*
- Greenberg, *Euclidean and Non-Euclidean Geometries*
- Martin, *Geometric Constructions*
- Coxeter, *Introduction to Geometry*
- Coxeter & Greitzer, *Geometry Revisited*
- Devaney, *An Introduction to Chaotic Dynamical Systems*
- Barnsley, *Fractals Everywhere*
- Devaney: *Iteration. Fractals. Chaos. The Mandelbrot Set.*