Walter Payton College Prep
Honors KAM I: Geometry Course Outline

Course Description
KAM I: Geometry is a college-level course similar to 100- or 200-level courses in advanced geometry (first semester) and dynamical systems (second semester). The course begins with topics in Euclidean geometry beyond what is covered in the standard Geometry curriculum: advanced constructions, triangle centers, the Fermat Point, the Nine Point Circle, etc. We then explore the algebra of transformations and include an in-depth look at synthetic inversion. In the second semester, we study axiomatic non-Euclidean Geometry, including Hilbert's axioms, Spherical, and Hyperbolic geometry. Finally, we work through an introduction to iteration, fractals, and chaos, including some advanced work with dynamical systems theory. Julia sets and the Mandelbrot are studied at the conclusion of the course. Both semesters involve writing rigorous proofs, weekly problem sets, and a few out-of-class projects of various lengths.

Course Objectives
At the end of the course, students will be able to

- Complete advanced constructions involving multiple steps or ideas
- Prove theorems in advanced geometry
- Use matrices, compass and straightedge, and dynamic geometry software to apply planar transformations
- Prove many "obvious" theorems using Hilbert's axioms
- Explain the relationship between Neutral, Spherical, and Hyperbolic geometry, giving examples of models for each and theorems true in one but not the other two
- Model situations with dynamical systems
- Explain and predict the long-term behavior of systems using fundamental theorems about iteration, dynamical systems, and chaos
- Give examples of fractals, explain how they are generated, and describe their properties qualitatively and quantitatively
- Describe iterations on the complex plane algebraically and geometrically, including stable and nonstable behavior
- Explain what the Mandelbrot and Julia sets are, how they are related to each other, how their properties relate to each other, and prove theorems about them
- Prove basic theorems of analysis using rigorous definitions of fundamental concepts
- Engage in original mathematical research and present findings at QED: Chicago's Youth Math Symposium

Course Topics Calendar

Weeks 1-4	Construction: Pólya's method, straightedge and compass constructions, problem-solving; Theorems about circles
Weeks 5-8	Advanced Geometry Theorems; work on independent research (QED projects)
Weeks 9-16	Transformations, Symmetry Groups, and Inversion
Weeks 17-20	Hilbert's Axioms, Neutral Geometry
Weeks 21-23	Euclidean and Non-Euclidean Geometries; Apollonius Project
Weeks 24-27	Iteration of real-valued functions; phase space
Weeks 28-30	Fractals and fractal geometry; Hausdorff dimension
Weeks 31-34	Chaos: definition of chaos, formal dynamical systems; isomorphisms between dynamical systems
Weeks 35-37	Complex dynamics; Mandelbrot and Julia sets

Resources Used

- Pólya, *Mathematical Discovery vol. I*
- Altshiller-Court, *College Geometry*
- Greenberg, *Euclidean and Non-Euclidean Geometries*
- Martin, *Geometric Constructions*
- Coxeter, *Introduction to Geometry*
- Coxeter & Greitzer, *Geometry Revisited*
- Devaney, *An Introduction to Chaotic Dynamical Systems*
- Barnsley, *Fractals Everywhere*
- Devaney: *Iteration. Fractals. Chaos. The Mandelbrot Set.*